Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

THE GEOMETRIC OLD AND NEW.

By DR. GEORGE BRUCE HALSTED.

1. Perhaps the newest geometry in English is by James Howard Gore, professor of mathematics in Columbian University, and published by Longmans, Green & Co., first edition December, 1898, second edition, 1899. Mr. Gore defines the straight in his §6, as follows: "A straight line is the shortest line between two points." But this presumes beforehand the measurement of all lines, while the soundest of geometries, Euclid, will not even attribute length to the simplest of curves, the circle, and our new mathematics knows of lines, real boundaries between two parts of a plane, to which the idea of length is inapplicable. Moreover, before the execution of any measurement there must be a measuring standard; but this is first given by the straight line, is in fact always *a sect*, a definite piece of a straight. Again the existence of a minimum is presumed; which is not evident; and of a single minimum; which is a subtle assumption.

In fact the operation of measuring a geometric magnitude we cannot effect, rigorously speaking, either for curves or curved surfaces. For, however little may be the parts of a curve, they do not cease to be curves, and consequently they cannot be compared with a sect by superposition or congruence; just as parts of a curved surface are not comparable with portions of a plane by superposition or congruence. A paradoxical assumption is in fact necessary, which destroys by itself the primitive idea of measurement, the application of a standard unit.

Thus the evaluation of the length of a curve represents not at all a measurement of the primary kind, and until explicit post-Euclidean assumptions have been made, we cannot even know what is to be meant by one line being shorter than another between the same two points.

2. Mr. Gore defines an angle in §30: "An angle is the difference in direction of two lines that meet." But the word 'direction' can only be given a meaning when the theory of parallels is presupposed. No one has ever been able to say when two different straights have the same direction without using parallelism. Direction, to be understood in any strict sense whatever, presupposes three fundamental geometric ideas, namely, straight line, angle, parallels.

3. In §52 in the last step of an attempted demonstration, we read: "this would give two straight lines joining P and P', *which is impossible.*" On the contrary, in Riemann's double elliptic geometry every two straight lines meet twice.

4. Under Parallel Lines, we read: "§59. Definition. Two straight lines are called Parallel when they lie in the same plane, and cannot meet, *nor approach each other*, however far they may be produced."

Mr. Gore's unfortunate interpolation is thus paraphrased in his next article: "Since parallel lines cannot approach each other, they are everywhere equally distant from each other." This assumes that a line which is everywhere equidistant from a straight line must be itself straight, but in Bolyai's geometry this line is a curve, the well-known *equidistantial*, while in Lobachevski one of the first theorems is: "The farther parallel lines are prolonged on the side of their parallelism, the more they approach one another."

In §62, in place of a proof for the fundamental theorem: "If two parallels are cut by a transversal the alternate angles are equal," Mr. Gore gives the flat *petitio principii*: "The lines AB and CD, being parallel, have the same direction. The lines EG and GH, being in one and the same straight line, are similarly directed. That is, the angles EGB and GHD have sides with the same direction; therefore the differences of their directions are equal." ! ! !

"§127. A circle is a plane figure"!

"§138. Two circumferences are tangent to each other when they are tangent to a straight line at the same point"! What about two circles with only one point in common, and the straight tangent to one at that point different from the straight tangent to the other at that point?

Page 64, "Up to the present time it has been assumed that any needful line or combination of lines could be drawn, and the question has not arisen as to the possibility of drawing these lines *with accuracy ! ! !*

In order to show that any required combination of lines, angles, or parts of lines or angles fulfilled the required conditions, principles were needed long before they could be demonstrated"! Mr. Gore has perhaps never looked into a copy of Euclid. Beyond his three postulates, hypothetical constructions are neither necessary nor admissible.

There is one good purpose that this book may subserve, that is to show how absolutely essential is a knowledge of non-Euclidean geometry.

Austin, Texas, 1900.

EXPRESSION OF RIEMANN'S *P* FUNCTION AS A DEFINITE INTEGRAL.

By W. E. HEAL.

Consider the differential equation of Riemann's P function, namely,

$$(x-a)(x-b)(x-c)\frac{d^2y}{dx^2}+(x-a)(x-b)(x-c)\left(\frac{1-\alpha-\alpha'}{x-a}+\frac{1-\beta-\beta'}{x-b}+\frac{1-\gamma-\gamma'}{x-c}\right)\frac{dy}{dx}+\left(\frac{\alpha\alpha'(a-b)(a-c)}{x-a}+\frac{\beta\beta'(b-c)(b-a)}{x-b}+\frac{\gamma\gamma'(c-a)(c-b)}{x-c}\right)y=0 \quad \ldots\ldots (1),$$